Physics of turbulent flow - Centrale Lyon 3rdYear - MSc
Tutorial subject - Monday 22 November 2021

Dispersion of contaminants

A pollutant (salinity, particle, heat) of concentration 200g/l is injected during a time $t = 0.01$s in the whole section of a circular pipe of diameter $D = 0.2$m, with a water flow of bulk velocity $U_d = 5$ m.s^{-1}. What is the concentration of this pollutant at $2000D$ downstream of the pipe?

The two following configurations will be considered to determine the variance of the Lagragian position of fluid particles, in order to solve this problem,

- free space dispersion by stationary, homogeneous turbulence
- longitudinal dispersion in a pipe

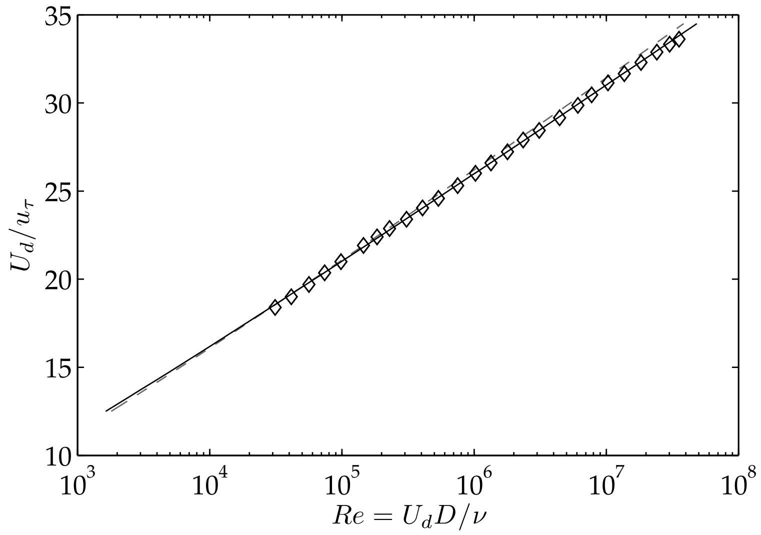

FIGURE 1 – Bulk velocity U_d normalized to the friction velocity u_τ plotted against the Reynolds number $\mathrm{Re}_D = U_d D/\nu$ for a circular pipe flow.[1]

—— from $1/C_f^{1/2} \simeq 3.860\log_{10}(\mathrm{Re}_D\, C_f^{1/2}) - 0.088$

– – – from $1/C_f^{1/2} \simeq 4\log_{10}(\mathrm{Re}_D\, C_f^{1/2}) - 0.40$

◊ data from McKeon *et al.* (Princeton group, superpipe)

A measurement of Lagrangian velocity autocorrelation in approximately isotropic turbulence

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(Received 12 January 1973)

By measuring the heat dispersion behind a heated wire stretched across a wind tunnel (Taylor 1921, 1935), the Lagrangian velocity autocorrelation was determined in an approximately isotropic, grid-generated turbulent flow. The techniques were similar to previous ones, but the scatter is less. Assuming self-preservation of the Lagrangian velocity statistics in a form consistent with recent measurements of decay in this flow (Comte-Bellot & Corrsin 1966, 1971), a stationary and an approximately self-preserving form for the dispersion were derived and approximately verified over the range of the experiment.

Possibly the most important aspect of this experiment is that data were available in the same flow on the simplest Eulerian velocity autocorrelation in time, the correlation at a fixed spatial point translating with the mean flow (Comte-Bellot & Corrsin 1971). Thus, the Lagrangian velocity autocorrelation coefficient function calculated from the dispersion data could be compared with this corresponding Eulerian function. It was found that the Lagrangian Taylor microscale is very much larger than the analogous Eulerian microscale (76 ms compared with 6·2 ms), contrary to an estimate of Corrsin (1963). The Lagrangian integral time scale is roughly equal to the Eulerian one, being larger by about 25 %.

Suivi de la propagation d'une nappe de pollution (simulation avec fluorescéine et Rhodamine B) sur la Garonne (2008)
https://www.smeag.fr/

Some measurements of particle velocity autocorrelation functions in a turbulent flow

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(Received 29 June 1970)

Particle velocity autocorrelations of single spherical beads (46·5 μ hollow glass, 87 μ glass, 87 μ corn pollen, and 46·5 μ copper) were measured in a grid-generated turbulence. The hollow glass beads were small and light enough to behave like fluid points; the other types had significant inertia and 'crossing trajectories' effects. The autocorrelations decreased much faster for heavier particles, in contradiction to previous experimental results. The integral scale for the copper beads was $\frac{1}{3}$ of that for the hollow glass beads. The particle velocity correlations and the Eulerian spatial correlation were coincident within experimental error when the separation was non-dimensionalized by the respective integral scale. The data generated by the hollow glass beads can be used to estimate Lagrangian fluid properities. The Lagrangian time integral scale is approximated by L/u', where L is the Eulerian integral scale and u' is the turbulence intensity.

References

[1] Bailly, C. & Comte Bellot, G., 2015, *Turbulence* (2nd Ed.), Springer, Heidelberg.

[2] Shlien, D.J. & Corrsin, S., 1974, A measurement of Lagrangian velocity autocorrelation in approximately isotropic turbulence, *J. Fluid Mech.*, **62**(2), 255-271.

[3] Snyder, W.H. & Lumley, J.L., 1971, Some measurements of particle velocity autocorrelation functions in a turbulent flow, *J. Fluid Mech.*, **48**(1), 41-71.

[4] Sullivan, P. J., 1971, Longitudinal dispersion within a two-dimensional turbulent shear flow, *J. Fluid Mech.*, **49**(3), 551-576.

[5] Taylor, G.I., 1954, The dispersion of matter in turbulent flow through a pipe, *Proc. Roy. Soc. London*, **A223**, 446-468.

HOMEWORK #7

Numerical simulation of the free space dispersion

We propose to generate a synthetic isotropic turbulent field, here in 2-D to ease laptop simulations, from random Fourier modes. This stochastic field can be used for the numerical simulation of the dispersion of particles in homogeneous turbulence. The (real) turbulent field $\boldsymbol{u}'$ is defined from the following Fourier integral,

$$\boldsymbol{u}'(\boldsymbol{x}) = \int \hat{\boldsymbol{u}}(\boldsymbol{k})\, e^{i\boldsymbol{x}\cdot\boldsymbol{k}} d\boldsymbol{k}$$

This relation is then discrete in N Fourier modes on the spectral half-space only, that is for $k_1 \geq 0$

$$\boldsymbol{u}'(\boldsymbol{x}) = 2\sum_{n=1}^{N} \tilde{u}_n \cos(\boldsymbol{k}_n \cdot \boldsymbol{x} + \psi_n)\, \boldsymbol{\sigma}_n \tag{1}$$

where $\tilde{u}_n$, ψ_n, $\boldsymbol{k}_n$ and $\boldsymbol{\sigma}_n$ are the amplitude, the phase, the wavenumber and the unit vector aligned with the spectral component of the n-th Fourier mode respectively, refer to Fig. 2. To generate a homogeneous velocity field, the probability density function (pdf) must be uniform over $0 \leq \psi < 2\pi$, and therefore $p(\psi) = 1/(2\pi)$. To satisfy the incompressibility condition $\nabla \cdot \boldsymbol{u}' = 0$, the unit vector is taken as follows, $\boldsymbol{\sigma} = \epsilon(-\sin\theta, \cos\theta)$ where ϵ is randomly chosen equal to ± 1. Finally, isotropy of the turbulent is obtained by prescribing for the pdf of θ,

$$p(\theta) = \frac{1}{\pi} \qquad \text{with} \qquad -\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$$

The amplitude of each mode is determined from the discretization of the turbulent kinetic energy spectrum $E(k)$,

$$k_t = \sum_{n=1}^{N} \tilde{u}_n^2 = \sum_{n=1}^{N} E(k_n)\, dk_n$$

where k_t is the turbulent kinetic energy. A Gaussian spectrum is selected here,

$$E(k) = \frac{3u_f^2}{k_0} \frac{k}{k_0} e^{-\left(\frac{k}{k_0}\right)^2}$$

where the scale of velocity fluctuations is given by $u_f = \sqrt{2k_t/3}$ and the integral length scale L_f with $k_0 L_f \simeq 1$. A simple linear distribution is chosen for the wavenumbers $k_n = (n/N)k_{\max}$ for $n = 1, \cdots, N$, where $k_{\max}$ is the highest wavenumber to be considered.

1. Generate a single realization of the synthetic field (1), and plot the 2-D velocity field on a domain $0 \leq x_1, x_2 \leq 2L_f$ where $L_f = 2\pi/k_0$, and with $N = 20$ and $k_{\max} = 2k_0$.
2. Verify numerically that $\overline{u_i' u_j'} \rightarrow u_f^2 \delta_{ij}$ when the number of realizations increases to perform the average.
3. Plot the two correlation functions $f(r)$ and $g(r)$.
4. Verify that the stochastic turbulent field is Gaussian by computing numerically the skewness and kurtosis factors.

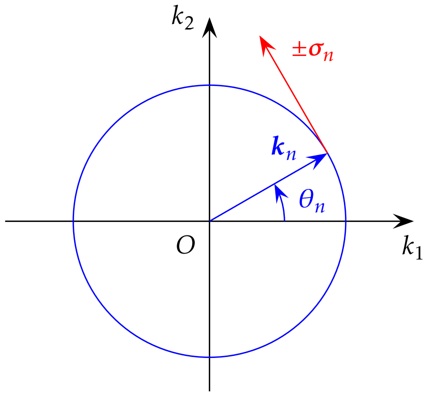

FIGURE 2 – Representation of the n-th Fourier mode in 2-D, see Eq. (1)

5. Application to the free space dispersion. By taking realistic parameter to generate the stochastic turbulent field, compute the distribution of a large number of contaminant particles by solving $d\boldsymbol{x}/dt = \boldsymbol{u}'$ with the same initial condition $\boldsymbol{x} = \boldsymbol{x}_0$ at time $t = 0$. Characterize the distribution of the cloud of particles, and verify that its variance is driven by the turbulent dispersion coefficient of Taylor. Any additional representation or discussion is welcome.