*L*aboratoire de *M*écanique des *F*luides et d'*A*coustique
LMFA UMR 5509

CENTRALE LYON

Physics of turbulent flow

Christophe Bailly

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Outline

Turbulent flows

- unsteady aperiodic motion
- unpredictable behaviour
- presence of a wide range of time and space scales

Turbulence appears when the source of the kinetic energy which drives the fluid motion is able to overcome viscosity effects, that is the Reynolds number must be sufficiently large

- astrophysics, geophysical flows including ocean circulation, climate, weather forecast, hydrology, dispersion of aerosols
- external aerodynamics for aeronautics & ground transportation, internal flows in mechanical engineering, biomechanics, biological flows
- noise of turbulent flows (aeroacoustics), sound propagation (atmosphere, ocean), fluid-solid interaction and vibroacoustics

Non-linearity of Navier-Stokes' equations

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right)=-\nabla p+\mu\nabla^2\boldsymbol{u}$$

The non-linear nature of the convective acceleration $\boldsymbol{u}\cdot\nabla\boldsymbol{u}$ is at the origin of the development of a large range of space and time scales, that are observed in a turbulent flow.

A (too) simple example illustrating the generation of harmonics is based on the simplified equation $\partial_t\boldsymbol{u}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}=0$, with $\boldsymbol{u}=(u_1,u_2)$ in 2-D. By assuming that at time t_0,

$$\begin{cases} u_1(x_1,x_2,t_0)=A\cos(k_1x_1)\sin(k_2x_2) \\ u_2(x_1,x_2,t_0)=B\sin(k_1x_1)\cos(k_2x_2) \end{cases}$$

with $Ak_1+Bk_2=0$ to satisfy the incompressibility condition $\nabla\cdot\boldsymbol{u}=0$

Non-linearity of Navier-Stokes' equations (cont.)

A Taylor series of the velocity $\boldsymbol{u}$ around t_0 provides

$$\boldsymbol{u}(\boldsymbol{x},t) = \boldsymbol{u}(\boldsymbol{x},t_0) + (t-t_0)\,\partial_t \boldsymbol{u}|_{t_0} + \ldots \qquad \text{with } \partial_t \boldsymbol{u}|_{t_0} = -\boldsymbol{u}\cdot\nabla\boldsymbol{u}|_{t_0}$$

As an illustration, one gets for u_1

$$\begin{aligned} u_1(x_1,x_2,t) &= A\cos(k_1x_1)\sin(k_2x_2) \\ &\quad + (t-t_0)\,\frac{k_1A^2}{2}\left[\cos(2k_1x_1)\sin^2(k_2x_2) + \sin(2k_1x_1)\cos^2(k_2x_2)\right] + \ldots \end{aligned}$$

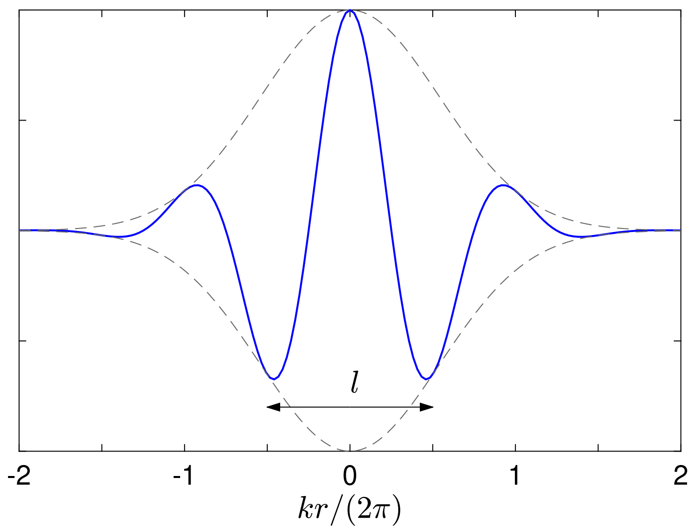
It can be noted the production of higher harmonics $(2k_1, 2k_2, k_1+k_2)$, that is of larger wavenumbers corresponding to smaller structures, and also of smaller harmonics (k_1-k_2)

What is a turbulent structure of wavenumber k ?

What is the smallest structure that can survive in the flow, before destruction by viscosity ?

Representation in spectral space

Model of a turbulent structure of wavenumber k : energy is contained in a narrow band around $k = 2\pi/l$, where l is a characteristic length scale, see figure on the right



$f(r) = \cos(kr)\exp(-\log(2)(r/r_0)^2)$ with $r_0 = 4/k$ here

The Fourier transform of f is centered around k

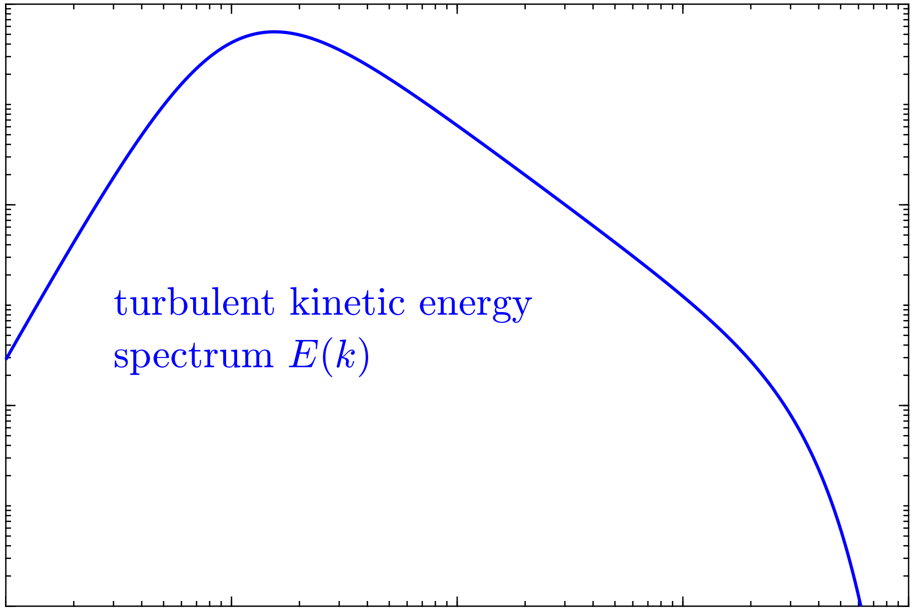
A real turbulent structure (vortex or eddy) can be decomposed into waves of different wavelengths, with their amplitude and phase, using Fourier transform

Various other decompositions can also be used (wavelets for instance)

A structure of wavenumber k (of size $\sim 1/k$) can be seen as an elementary component of the previous decomposition

Viscous scales

The energy transfer induced by the convective acceleration $\boldsymbol{u} \cdot \nabla \boldsymbol{u}$ is stopped by the molecular viscosity (impossible to preserve small structures with too large velocity gradient)



Smallest structures u_η $k_\eta \sim 1/l_\eta$

$$\frac{\partial \boldsymbol{u}}{\partial t} \simeq \nu \nabla^2 \boldsymbol{u} \quad \text{(Stokes)}$$

The balance between the two terms

$$\frac{\partial \boldsymbol{u}}{\partial t} \sim \frac{u_\eta^2}{l_\eta} \qquad \nu \nabla^2 \boldsymbol{u} \sim \nu \frac{u_\eta}{l_\eta^2}$$

leads to

$$\mathrm{Re}_\eta = \frac{l_\eta u_\eta}{\nu} \sim 1$$

These viscous scales (u_η, l_η), also called Kolmogorov's scales, are the smallest scales of the flow allowed by viscosity. They impose the spatial resolution necessary for measurement or simulation

- **Turbulence is part of continuum mechanics**
  Viscous scale l_η wrt the free mean path λ_l of molecules

Knudsen number $$\mathrm{Kn} = \frac{\lambda_l}{l_\eta} \ll 1$$

- **Sensitivity to initial conditions**

  The nonlinearity of the Navier-Stokes equations does not allow the time evolution of turbulent fields to be predicted over a long period. The reason for this is that a small difference in the initial conditions introduces significant differences as time goes, linked to the largest Lyapunov exponent for chaotic systems.

  An initial separation of 1 cm between two fluid particles in the atmosphere results in a 10 km separation within just a day, the butterfly effect in chaos theory !

  Ruelle, D. and Takens, F., 1971, On the nature of turbulence, *Commun. Math. Phys.*, **20**, 167–192

Mean and fluctuating quantities

The statistical mean $\bar{F}(\boldsymbol{x},t)$ of a variable $f(\boldsymbol{x},t)$ is defined as

$$\bar{F}(\boldsymbol{x},t) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} f^{(i)}(\boldsymbol{x},t)$$

where $f^{(i)}$ is the i-th realization : convenient when manipulating equations but difficult to implement in practice, or even impossible for geophysical flows !

Can we approximate the ensemble mean $\bar{F}$ of $f = \bar{F} + f'$ by a sufficiently long time average F_T of one realization only ?

$$F_T = \frac{1}{T}\int_0^T f(t)dt$$

Time average

Time average makes sense only if turbulence is stationary, that is statistics are independent of time. The autocorrelation coefficient $\mathcal{R}$ is then only an even function of the time separation τ

$$\mathcal{R}(\tau) = \frac{\overline{f'(t)f'(t+\tau)}}{\overline{f'^2}}$$

We can estimate the difference between F_T obtained by a finite integration time and the true (ensemble) mean value $\bar{F}$ by considering

$$F_T - \bar{F} = \frac{1}{T}\int_0^T \left[f(t) - \bar{F}\right] dt = \frac{1}{T}\int_0^T f'(t)\,dt$$

The mean square value is

$$(F_T - \bar{F})^2 = \frac{1}{T}\int_0^T f'(t_1)\,dt_1 \times \frac{1}{T}\int_0^T f'(t_2)\,dt_2$$

Time average (cont.)

By taking the statistical average, that is $\overline{(F_T - \bar{F})^2}$, one has

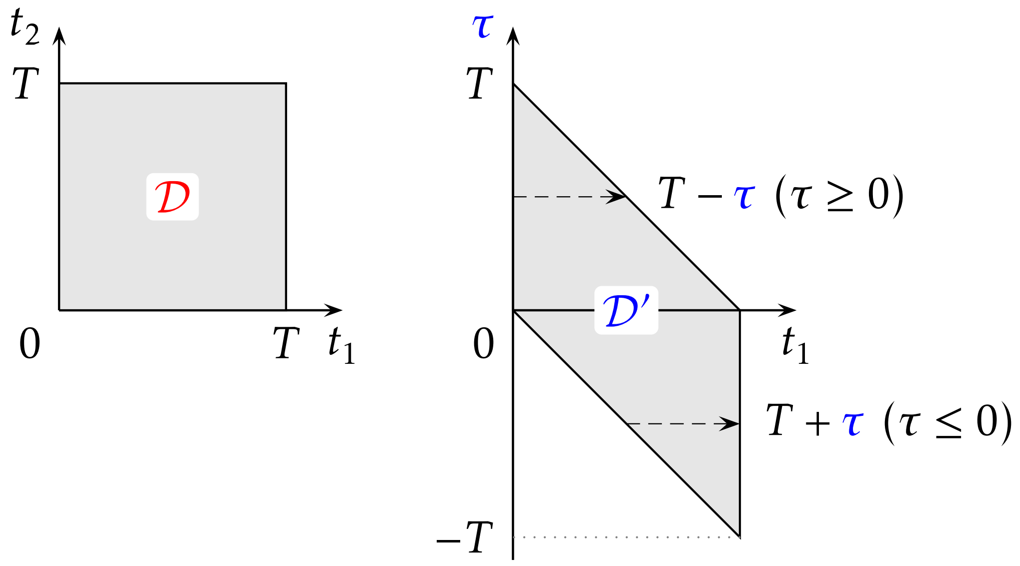
$$\overline{(F_T - \bar{F})^2} = \frac{\overline{f'^2}}{T^2} \iint_{\mathcal{D}} \mathcal{R}(t_2 - t_1)\, dt_1\, dt_2 = \frac{\overline{f'^2}}{T^2} \iint_{\mathcal{D}'} \mathcal{R}(\tau)\, dt_1\, d\tau \qquad \tau = t_2 - t_1$$

The integration over t_1 can be achieved by splitting the domain $\mathcal{D}'$ as follows,

$$\iint_{\mathcal{D}'} \mathcal{R}(\tau) dt_1 d\tau$$

$$= \int_0^T (T-\tau)\mathcal{R}(\tau)d\tau + \int_{-T}^0 (T+\tau)\mathcal{R}(\tau)d\tau$$

$$= 2\int_0^T (T-\tau)\mathcal{R}(\tau)d\tau$$



Time average (cont.)

The mean square error between F_T and the true mean value $\bar{F}$ can thus be estimated as

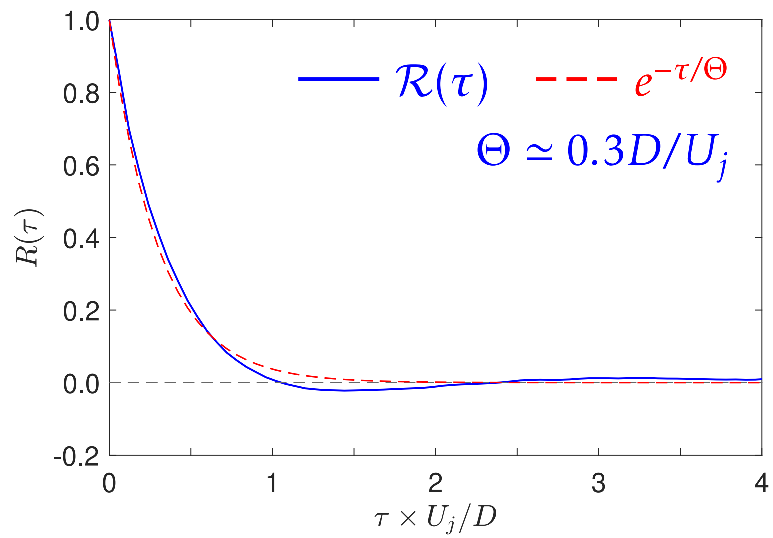
$$\overline{(F_T-\bar{F})^2} = 2\,\frac{\overline{f'^2}}{T}\int_0^T\left(1-\frac{\tau}{T}\right)\mathcal{R}(\tau)\,d\tau \simeq 2\,\frac{\overline{f'^2}}{T}\int_0^T\mathcal{R}(\tau)\,d\tau \simeq 2\overline{f'^2}\,\frac{\Theta}{T}$$

if the time integration T is much longer than the integral time scale Θ, defined by

$$\Theta = \int_0^{\tau^\star}\mathcal{R}(\tau)d\tau$$

where $\tau^\star = \infty$ or the first zero crossing of $\mathcal{R}(\tau)$ in practice

The term τ/T is then small in the range of τ where $\mathcal{R}(\tau)$ is non-zero, and the time average value $F_T \to \bar{F}$ as $T \to \infty$



Subsonic jet at $\mathrm{Re}_D = 10^5$
$x_1 = 0,\ x_2 = 2D$

- **Ergodicity**

  By considering the time average in signal processing to approximate the ensemble mean, we assume that turbulence is an ergodic process.

  Ergodicity expresses the idea that a trajectory of a dynamical system (of a stochastic process signal) will eventually visit all parts of the phase space in which the system moves, in a uniform and random direction. Statistical properties can thus be deduced from a single (sufficiently long) realization.

Textbooks

Batchelor, G.K., 1967, An introduction to fluid dynamics, *Cambridge University Press*, Cambridge.

Bailly C. & **Comte Bellot** G., 2003 Turbulence, *CNRS éditions*, Paris (out of print).
———, 2015, *Turbulence* (in english), Springer, Heidelberg.
(360 pages, 147 illustrations, Foreword by Charles Meneveau)

Bailly C. & Comte Bellot G., 2003, *Turbulence* (in french), CNRS éditions, Paris.
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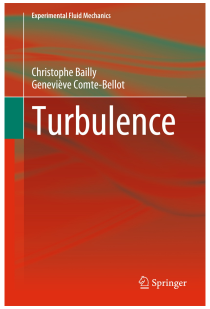

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Also *Pergamon Press*, 2nd edition, 1987.

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White F., 2005, Viscous fluid flow, 3ed Ed., *McGraw-Hill, Inc.*, New-York (1st Ed. 1974).

- **Outline**

  The main objectives are the mastery of basic concepts (turbulence production, turbulence boundary layer, role of vorticity, homogeneous and isotropic turbulence, Kolmogorov theory), the development of skills in turbulence modeling, the critical analysis of results, and the acquisition of a global vision of experimental approaches.

  - Introduction
  - Statistical description of turbulent flows
  - Wall-bounded turbulent flows
  - Dynamics of vorticity
  - Homogeneous and isotropic turbulence
  - Dynamics of isotropic turbulence – Kolmogorov's theory
  - Introduction to experimental techniques

Outline (cont.)

- **Practical work**

  Lab-session Numerical simulation of the mean flow in a channel
  BE1 – Small class of 4 hours - exercices
  BE2 – Small class of 4 hours to solve a complete problem

  Auditors : you are invited to follow these practical activities
  (Let us know about it !)

- **Teaching team** Christophe Bailly
  Christophe Bogey

Assessment for this course

- There are one practical lab session, and two small classes of 4h (so-called 'BE', may involve signal processing, coding of simple models using Matlab and analytical developments). **For 3rd year students,** the grade is obtained with BE 60% and lab work 40%.

- Absence : it is possible to exceptionally modify a lab session, only by exchanging your session with that of another student.

- **Master student**, additional final exam (closed book and open notes), wednesday 18 december 2024. The final mark will be the max between – the final exam mark – and (50% final exam + 30% BE + 20% lab work).

- **Course slides** can be downloaded by following this link https://acoustique.ec-lyon.fr/christophe.bailly.php#turbulence

airfoil	profil
bluff body	corps non profilé
boundary layer	couche limite
bulk velocity	vitesse de débit
buoyancy	flottabilité
curl	rotationnel
chord	corde
conservative force	force qui dérive d'un potentiel (gravité par exemple)
creeping flow	écoulement rampant
Darcy friction coefficient	coefficient de pertes de charge
drag	traînée
density (mass per unit volume)	masse volumique
efficiency	rendement
energy head	charge
friction velocity	vitesse de frottement
head loss	perte de charge
inviscid flow	écoulement non visqueux
leading edge	bord d'attaque (d'un profil)
lift	portance
lift-to-drag ratio	finesse
mass fraction	fraction massique
mixture	mélange
point vortex	tourbillon ponctuel

relative density	densité
shaft work	travail de l'arbre (d'une machine tournante)
skin-friction coefficient	coefficient de frottement
slip boundary condition	condition aux limite glissante
stall	décrochage
strain (deformation) tensor	tenseur des déformations
stream function	fonction de courant
streamlined body	corps profilé
stress tensor	tenseur des contraintes
thrust	poussée
torque (angular momentum)	couple
trailing edge	bord de fuite (d'un profil)
vortex shedding frequency	fréquence du lâcher tourbillonnaire
vortex sheet	nappe (infiniment mince) de vorticité
wake	sillage
wall shear stress	contrainte pariétale

aka	also known as
wrt	with respect to

- **Both indicial and boldface notations are used to indicate vectors**

vector $\boldsymbol{U} \equiv \overrightarrow{U}$, i-th component U_i, norm U, $U^2 = \boldsymbol{U} \cdot \boldsymbol{U}$
gravity $\boldsymbol{g}$, $g_i = -g\delta_{3i}$, $\boldsymbol{g} = (g_1, g_2, g_3) = (0, 0, -g)$, $g = 9.81$ m.s^{-2}

density ρ (kg.m^{-3})
δ_{ij} Kronecker delta

Einstein summation convention
When an index variable appears twice in a single term (dummy index), it implies summation of that term over all the values of the index.

Scalar product between two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$

$$\boldsymbol{a} \cdot \boldsymbol{b} = \sum_{i=1}^{3} a_i b_i = a_i b_i \quad \text{(dummy index } i \text{ repeated)}$$

Short quiz $\quad \delta_{ij} a_j = ? \quad \delta_{ij}\delta_{ij} = ?$

- **Differential operators (expressed in Cartesian coordinates here)**
  The dot symbol · is never decorative : scalar product

Gradient

$$\boldsymbol{b} = \nabla f \equiv \overrightarrow{\text{grad}}\, f \qquad b_i = \frac{\partial f}{\partial x_i}$$

Divergence

$$\nabla \cdot \boldsymbol{U} = \text{div}(\boldsymbol{U}) = \sum_{i=1}^{3} \frac{\partial U_i}{\partial x_i} = \frac{\partial U_i}{\partial x_i}$$

Laplacian

$$\nabla^2 f = \Delta f = \sum_{i=1}^{3} \frac{\partial^2 f}{\partial x_i \partial x_i} = \frac{\partial^2 f}{\partial x_i \partial x_i}$$

Curl

$$\nabla \times \boldsymbol{U} = \overrightarrow{\text{rot}}\, \boldsymbol{U}$$

Differential operators (cont.)

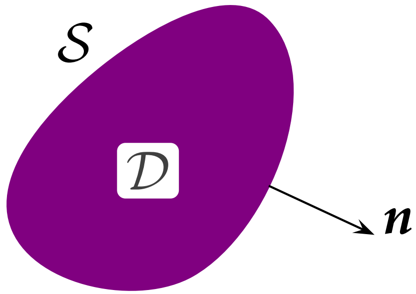
Explicit expression of the velocity gradient tensor $\nabla \boldsymbol{U}$

$$\nabla \boldsymbol{U} = \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{x}} = \begin{pmatrix} \frac{\partial U_1}{\partial x_1} & \frac{\partial U_1}{\partial x_2} & \frac{\partial U_1}{\partial x_3} \\ \frac{\partial U_2}{\partial x_1} & \frac{\partial U_2}{\partial x_2} & \frac{\partial U_2}{\partial x_3} \\ \frac{\partial U_3}{\partial x_1} & \frac{\partial U_3}{\partial x_2} & \frac{\partial U_3}{\partial x_3} \end{pmatrix}$$

Differential operators (cont.)

Divergence theorem : the involved surface is a closed surface (domain $\mathcal{D}$ bounded by the surface $\mathcal{S}$ and $\boldsymbol{n}$ unit outward normal vector)

$$\int_{\mathcal{D}} \nabla \cdot \overline{\overline{\boldsymbol{A}}}\, dv = \int_{\mathcal{S}} \overline{\overline{\boldsymbol{A}}} \cdot \boldsymbol{n}\, ds$$



for any given tensor $\overline{\overline{\boldsymbol{A}}}$.

As an illustration, one has for the pressure term :

$$\int_{\mathcal{D}} \nabla p\, dv = \int_{\mathcal{D}} \nabla \cdot (p\overline{\overline{\boldsymbol{I}}})\, dv = \int_{\mathcal{S}} p\overline{\overline{\boldsymbol{I}}} \cdot \boldsymbol{n}\, ds = \int_{\mathcal{S}} p\boldsymbol{n}\, ds$$