Small class #2

Expected work : you must write a report (handwritten or typed) with your answers to the questions and your personal comments (the most important). It is possible to submit one report per pair of students. To be submitted on moodle by the evening of December 20, 2024.

Thermal turbulence

Temperature fluctuations θ are considered with the Reynolds decomposition for temperature $T = \bar{T} + \theta$ and velocity $u_i = \bar{U}_i + u_i'$, in the framework of an incompressible turbulent flow. The buoyancy force is neglected here. The starting point of this exercise is the following simplified transport equation for temperature

$$\frac{\partial T}{\partial t} + \frac{\partial (u_i T)}{\partial x_i} = \alpha \frac{\partial^2 T}{\partial x_j \partial x_j} \tag{1}$$

1. Remind briefly the assumptions used to derive this equation, the expression and unit of α as well.
2. Write the non conservative form of Eq. (1) (but don't use it !).
3. Following what was done in the first small class, write the transport equation for the temperature fluctuation θ.
4. Deduce that the transport equation for the mean square fluctuations $k_\theta = \overline{\theta^2}/2$ can be recast in the following form

$$\frac{\partial k_\theta}{\partial t} + \bar{U}_i \frac{\partial k_\theta}{\partial x_i} = -\overline{u_i'\theta} \frac{\partial \bar{T}}{\partial x_i} - \alpha \overline{\frac{\partial \theta}{\partial x_i} \frac{\partial \theta}{\partial x_i}} - \frac{1}{2} \frac{\partial}{\partial x_i} \overline{u_i' \theta^2} + \alpha \frac{\partial^2 k_\theta}{\partial x_j \partial x_j} \tag{2}$$

   Identify the different terms in a few words.
5. Simplify the previous equation for the homogeneous turbulence case.
6. The notations $\mathcal{P}_\theta$ and ϵ_θ are introduced to denote the production and the sink term of temperature fluctuations. The latter is referred to as the dissipation term, by analogy with the turbulent kinetic energy equation.

   Which terms in equation (2) do they correspond to ? Provide the units and the sign of these terms.
7. In order to close this equation, the correlation term $-\overline{u_i'\theta}$ needs to be modeled. Remind the definition of the Prandtl number σ. Following what has been done for Reynolds' tensor in introducing ν_t, propose a closure for the term $-\overline{u_i'\theta}$ by introducing the turbulent Prandtl number $\sigma_t = \nu_t/\alpha_t$.
8. What is the sign of the term $\mathcal{P}_\theta$ in the framework of this model ? Comment your answer.

References

[1] Corrsin, S., 1951, The decay of isotropic temperature fluctuations in an isotropic turbulence, *J. Astronaut. Sci.*, **18**(6), 417-423.

[2] Corrsin, S., 1951, On the spectrum of isotropic temperature fluctuations in an isotropic turbulence, *J. Appl. Phys.*, 22(4), 469-473.

[3] Corrsin, S., 1953, Interpretation of viscous terms in the turbulent kinetic energy, *J. Astronaut. Sci.*, **20**(12), 853-854.

[4] Tennekes, H. & Lumley, J.L., 1972, A first course in turbulence, *MIT Press*, Cambridge, Massachussetts.

Oscillating boundary layer

Consider an incompressible boundary layer developing along a flat plate,[1,2] and an imposed freestream (external) velocity given by

$$U_{e1} = U_0 \{1 - a x_1 [1 - \cos(2\pi f_e t)]\}$$

where a is a constant, dimension of inverse length, U_0 is a constant reference velocity and f_e is the oscillation frequency.

1. Compute the pressure gradient dp_e/dx_1 in the external flow, and its mean value $d\bar{P}_e/dx_1$ by integrating over one period $T_e = 1/f_e$.

2. Derive the expression of dp_e/dx_1 for the steady case $U_{e1} = U_0(1 - a x_1)$, without oscillation.

3. What conclusion can you draw about a possible separation of the boundary layer ?

References

[1] Fan, S., Lakshminarayana, B. & Barnett, M., 1993, Low-Reynolds-number $k - \epsilon$ model for unsteady turbulent boundary-layer flows, *AIAA Journal*, **31**(10), 1777-1784.

[2] Wilcox, D. C., 1988, Multiscale model for turbulent flows, *AIAA Journal*, **26**(11), 1311-1320.