*L*aboratoire de *M*écanique des *F*luides et d'*A*coustique
LMFA UMR 5509

ÉCOLE CENTRALE LYON

Physics of turbulent flow : exercises

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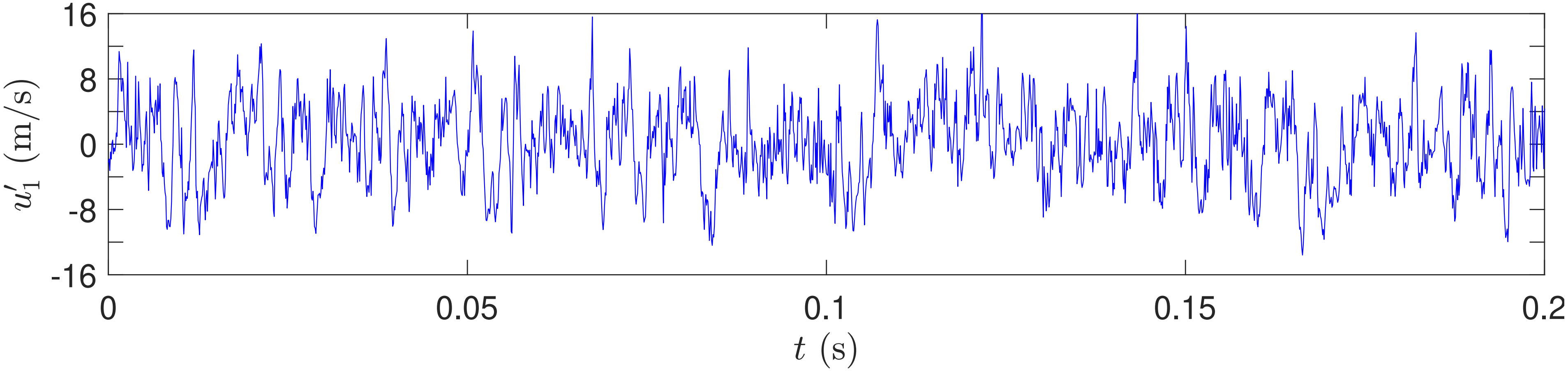
http://acoustique.ec-lyon.fr

- **Fluctuating velocity signal in the shear layer of a subsonic round jet**
(measured by crossed-wire probes at $x_1 = 2D$, $x_2 = D/2$, $x_3 = 0$)

Nozzle diameter $D = 50$ mm, exit velocity $U_j = 30$ m.s^{-1}
$\rightsquigarrow$ Reynolds number $\mathrm{Re}_D = 10^5$

$u_1'(t)$

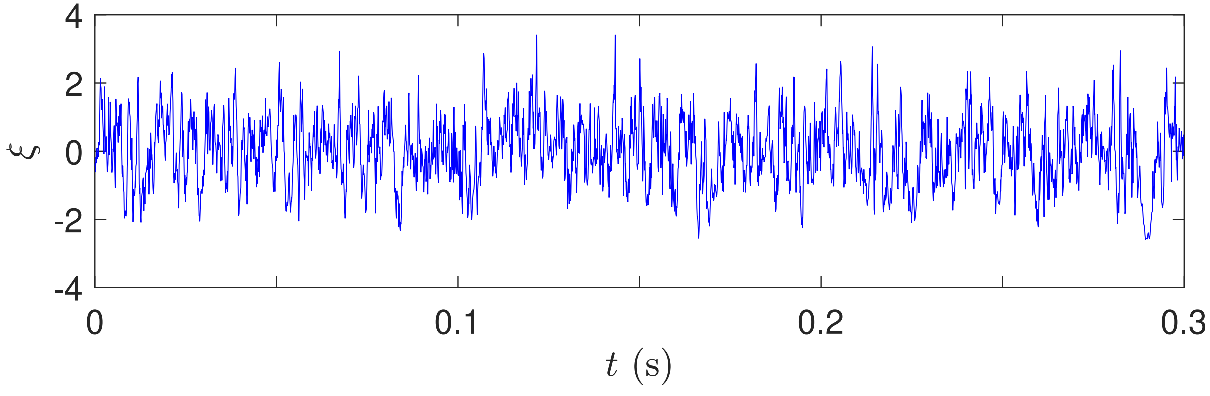


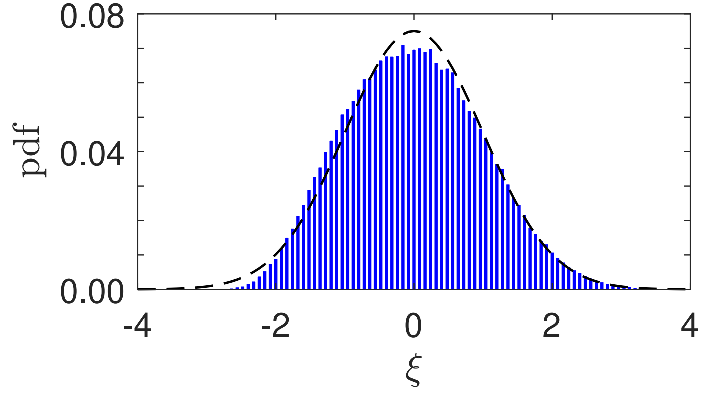
Courtesy of Emmanuel Jondeau (LMFA) for the data

Fluctuating velocity signal in the shear layer of a jet (cont.)

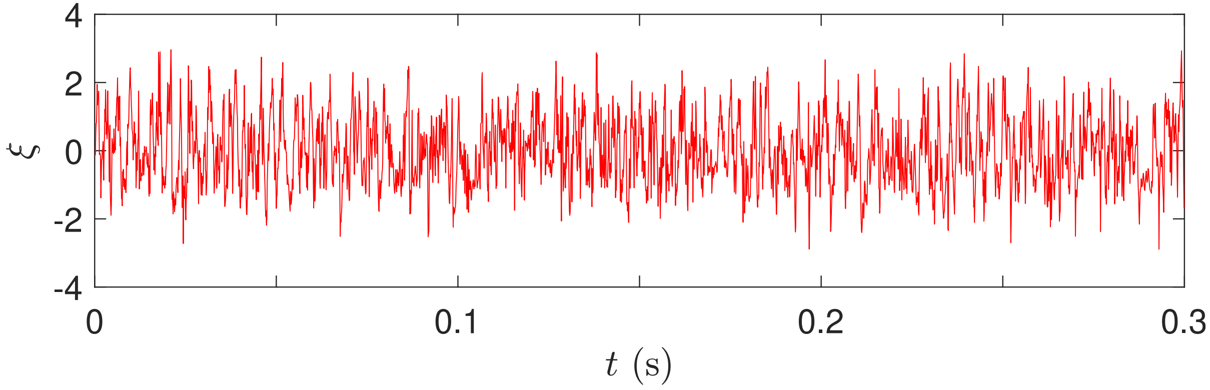
$u_1'(t)$ and $u_2'(t)$ with $\xi = u_\alpha'(t)/u_{\alpha\mathrm{rms}}$

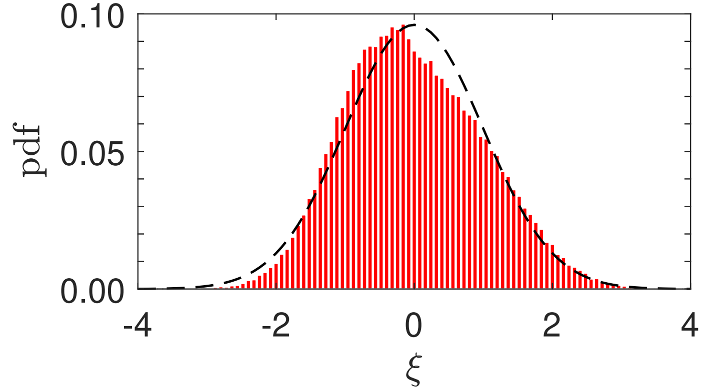
$S_{u_1} \simeq 0.21$ $T_{u_1} \simeq 2.74$





$S_{u_2} \simeq 0.24$ $T_{u_2} \simeq 2.76$





Interpretation ?

Probability density function (pdf)

For a centered variable $x_i' \equiv x_i - \bar{X}_i$ of root-mean-square deviation $x'_{i,\text{rms}} = \sigma_{x_i}$, the skewness S and the flatness or kurtosis T factors are defined by

$$S_{x_i} \equiv \frac{\overline{x_i'^3}}{x'^3_{i,\text{rms}}} \qquad\qquad T_{x_i} \equiv \frac{\overline{x_i'^4}}{x'^4_{i,\text{rms}}}$$

Skewness is a measure of the asymmetry of the pdf about its mean, and flatness is a measure of the tailedness of the pdf.

The reference is the Gaussian (normal) distribution, the pdf is

$$p(\xi) = \frac{1}{\sqrt{2\pi}\sigma_{x_i}} \exp\left(-\frac{\xi^2}{2\sigma^2_{x_i}}\right) \qquad x'_{i,\text{rms}} = \sigma_{x_i} \qquad S_{x_i} = 0 \qquad T_{x_i} = 3$$

$$1 = \int_{-\infty}^{+\infty} p(\xi)d\xi \qquad \overline{x_i'^n} = \int_{-\infty}^{+\infty} \xi^n p(\xi)d\xi$$

Probability density function (cont.)

$$\int_{\xi_1}^{\xi_2} p(\xi)\,d\xi = \frac{\text{nbr of expts in which } \xi_1 \leq x'_i \leq \xi_2}{\text{total number of expts}}$$

Reynolds stress tensor in the shear layer

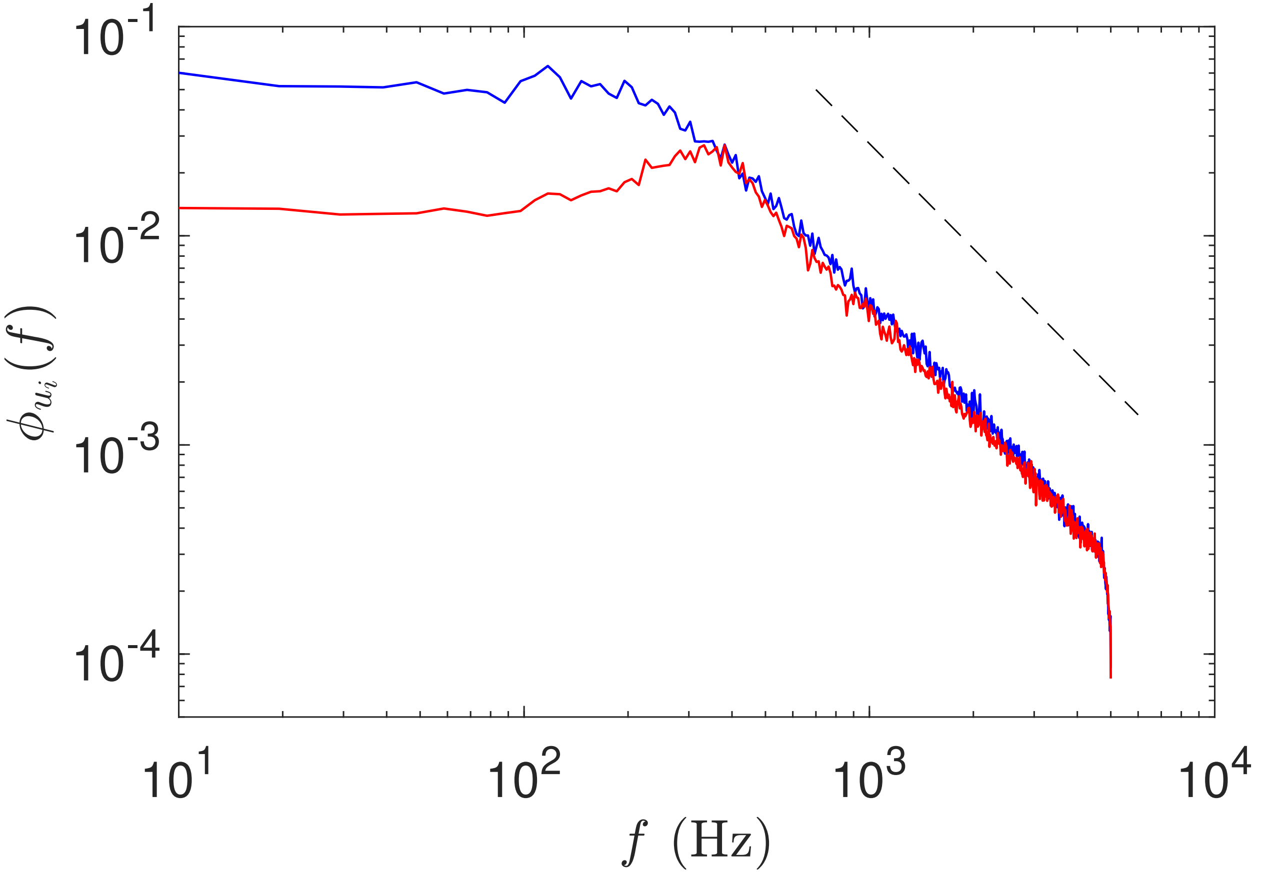
$$\frac{\sqrt{|\overline{u_i' u_j'}|}}{U_j} \simeq \begin{pmatrix} 0.18 & 0.10 & 0.10 \\ 0.10 & 0.14 & - \\ 0.10 & - & 0.13 \end{pmatrix}$$

- $\overline{u_1' u_2'} > 0$, in agreement with Boussinesq's hypothesis $-\overline{u_1' u_2'} = \nu_t \partial \overline{U}_1 / \partial x_2$ (okay, at least for the sign !)

- The Schwarz inequality is (must be !) satisfied, $|\overline{u_1' u_2'}|^2 \leq \overline{u_1'^2} \times \overline{u_2'^2}$

- Turbulence intensity $u'_{1,\mathrm{rms}}/U_j \simeq 0.18$

- **Spectra of u'_1 and u'_2**
  $f_s = 1/\Delta t = 10^4$ Hz, $n_{\text{fft}} = 1024$, $\Delta f = f_s/n_{\text{fft}}$, $f_{\max} = f_s/2 = 5$ kHz
  ($l_\eta \simeq 2.4 \times 10^{-5}$ m, $f_\eta = 2.6 \times 10^4$ Hz)



$$\int_0^\infty \phi_{u_1}(f)\,df = u'^2_{u_1,\text{rms}} \qquad \text{(Parseval)}$$

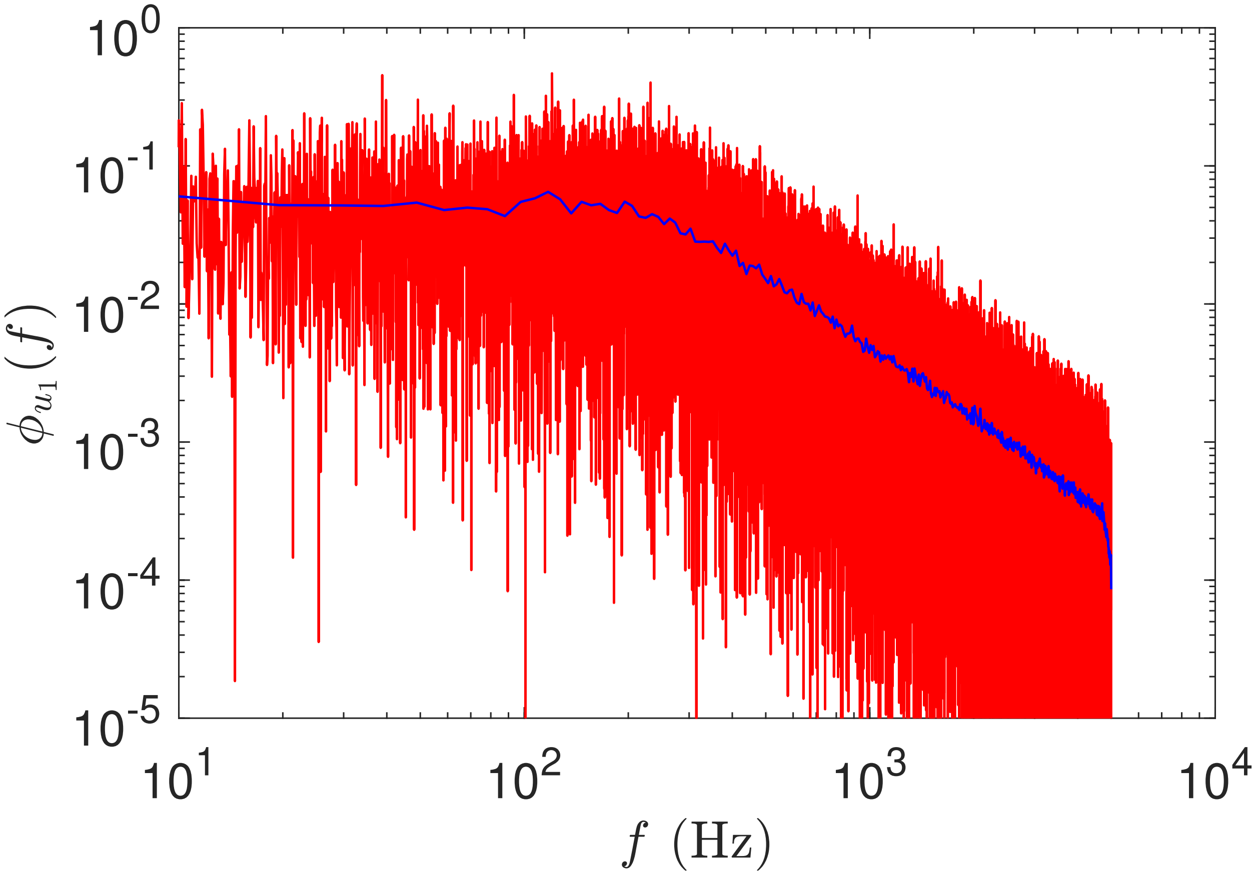
Spectra of u_1' and u_2'

```
%.. number of points for the Fourier transform
nfft = 1024;

dt = t(2)-t(1);
fs = 1./dt;

%.. Power spectral density
window = ones(nfft,1);
noverlap = nfft/2;

[PSDu f] = pwelch(u,window,noverlap,nfft,1./dt);
PSDu_int = trapz(f,PSDu);

disp(['PSD - Welch E = ',num2str(PSDu_int)]);
disp(['df = ',num2str(1/(nfft*dt)),' fmax = ',num2str(1/(2*dt))]);
```

- **Do not confuse a single fft and the power spectral density estimation**

FFT^2 of $u_1'(t)$ over $n_{\mathrm{fft}} = 150000$ points (with all the signal points)

FFT^2 $\neq$ PSD power spectral density $\phi_{u_1 u_1}(f)$!



- **Power spectral density for a stationary random process**
  (signal of infinite energy)

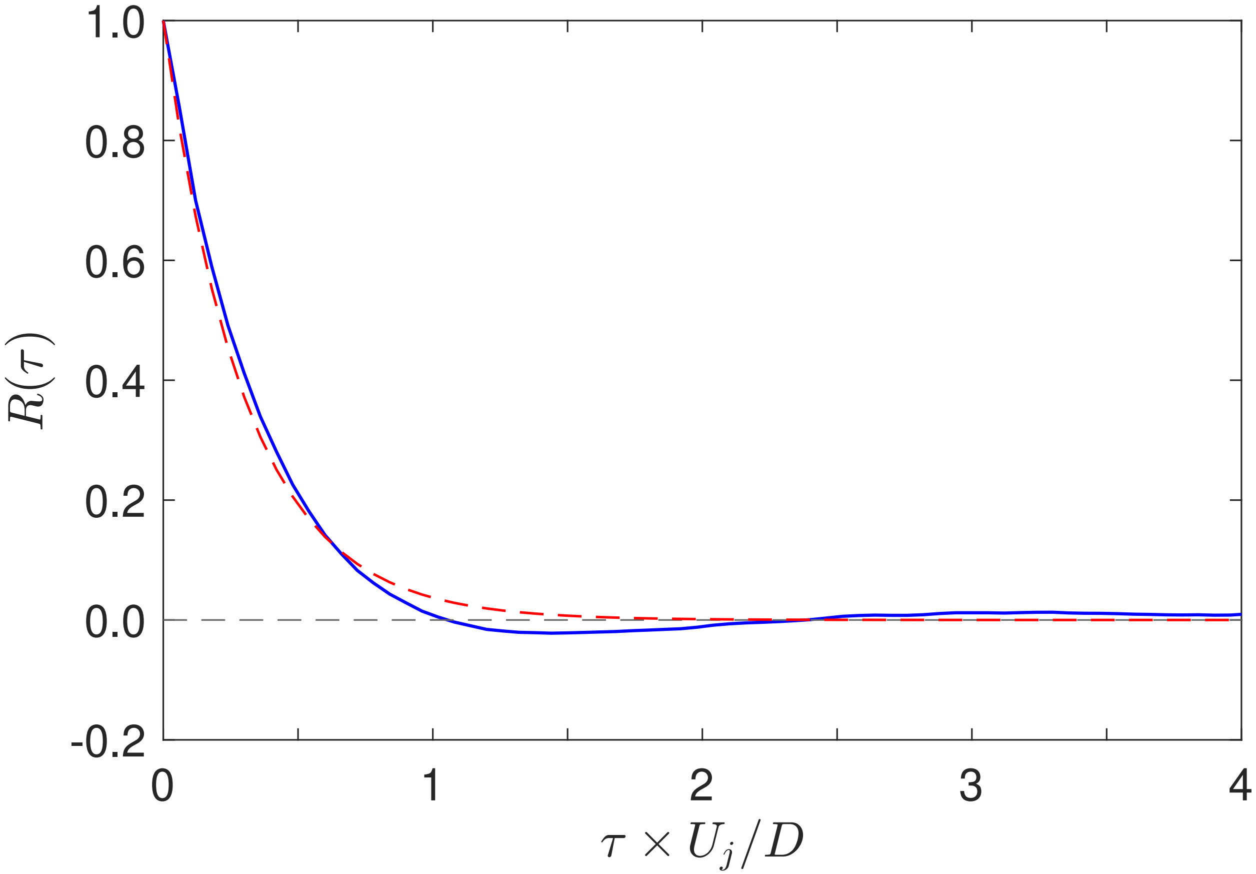
  Fourier transform calculated for a truncated signal of duration T
  $\hat{u}_1(f,k)$ is the DFT of $u_1(t)$ for the k-th block, n_K block averages

  One-sided power spectral density ($f > 0$)

$$\phi_{u_1}(f) = \lim_{T\to\infty} \frac{2}{T} E\left[\hat{u}_1^\star(f,k)\,\hat{u}_1(f,k)\right] \simeq \frac{2}{n_K T}\sum_{k=1}^{n_K} \hat{u}_1^\star(f,k)\,\hat{u}_1(f,k)$$

$$\int_0^\infty \phi_{u_1}(f)\,df = u'^2_{u_1,\text{rms}} \qquad \text{(Parseval)}$$

Autocorrelation function of u_1'



$$R(\tau) = \frac{\overline{u_1'(t)u_1'(t+\tau)}}{u'^2_{1\mathrm{rms}}}$$

— $R(\tau)$

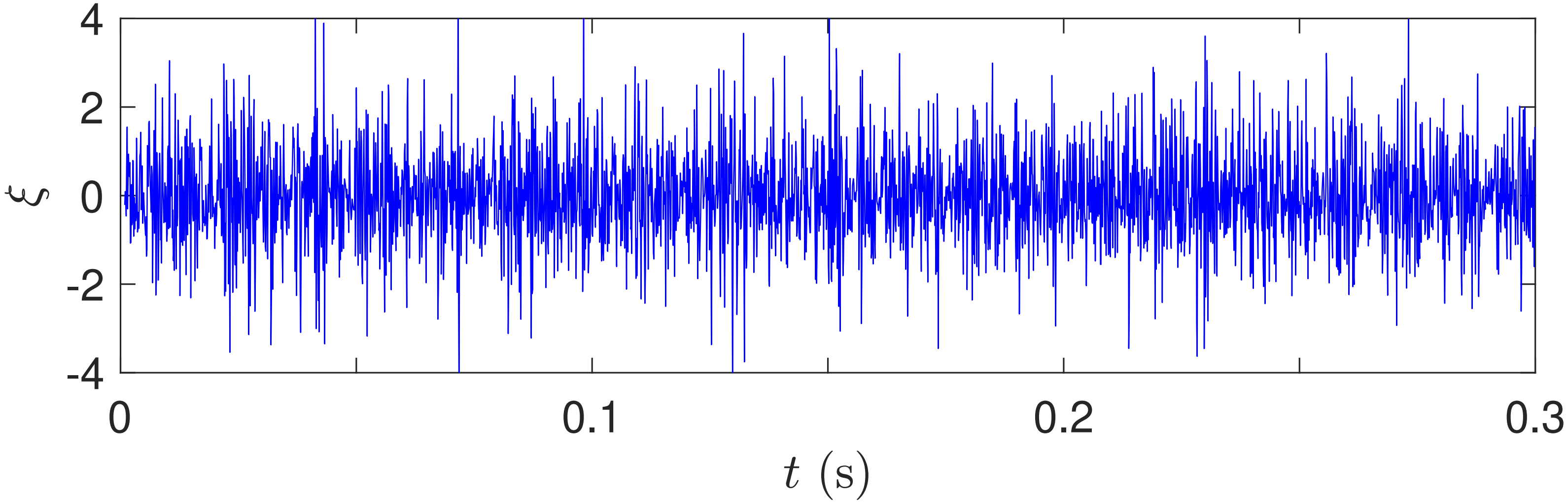
- - - $\exp(-\tau/\Theta)$

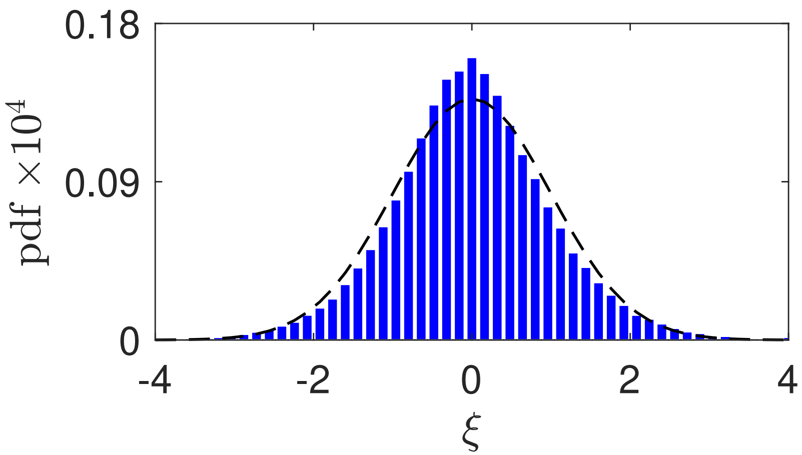
$$\Theta = \int_0^{\tau^\star} R(\tau)\,d\tau$$

$\Theta \simeq 5 \times 10^{-4}$ s, $\Theta \simeq 0.3 D/U_j$ (the integral time Θ is not the turbulence lifetime) and the corresponding integral space length $L_f \simeq \overline{U}_1 \times \Theta \simeq 1.5 \times 10^{-2}$ m

Time derivative of u_1'

$$\xi = \partial_t u_1' / \partial_t u_1'\big|_{\rm rms} \qquad S_{\partial_t u_1'} \simeq 0.07 \qquad T_{\partial_t u_1'} \simeq 4.03$$





Spectrum of du_1'/dt

can be computed from the signal du_1'/dt (using a finite difference scheme, *e.g.* 11-pt stencil, 4th order),

can be directly obtained from the spectrum of u_1' using $\phi_{\partial_t u_1'}(f) = (2\pi f)^2 \phi_{u_1'}(f)$