Homework #5 (Exam 2018)

Only the two fisrt exercises must be considered for the Homework #5

The notations used are those of the course. The arguments and comments associated with the established results should be carefully presented. The arguments must be short and the different parts of the subject are independent.

Relation of Corrsin & Kistler (1954) (6 pts)

In this exercice, the necessary rotational feature of a turbulent velocity field is emphasized.[1]

1. Remind the definition of an irrotational flow.
2. By considering the following quantity
$$\overline{u_i'\left(\frac{\partial u_i'}{\partial x_j} - \frac{\partial u_j'}{\partial x_i}\right)}$$
demonstrate that
$$\frac{\partial}{\partial x_i}\overline{u_i'u_j'} = \frac{\partial k_t}{\partial x_j}$$
3. Deduce the form of the RANS Equation for the case of a fluctuating irrotational velocity field, and comment carefully your result.
4. Is the Reynolds tensor diagonal for an irrotational flow?

Thermal turbulence (10 pts)

Temperature fluctuations θ are considered with $T = \bar{T}+\theta$. The starting point of this exercice is the following transport equation for the temperature

$$\frac{\partial T}{\partial t} + \frac{\partial (u_i T)}{\partial x_i} = \alpha \frac{\partial^2 T}{\partial x_j \partial x_j}$$

1. Remind briefly the assumptions used to derive this equation, and the unit of $\alpha \equiv \lambda/(\rho c_p)$.
2. Write the transport equation for the temperature fluctuation θ.
3. Deduce that the transport equation for the mean square fluctuation $k_\theta = \overline{\theta^2}/2$ can be recast in the following form
$$\frac{\partial k_\theta}{\partial t} + \overline{U}_i \frac{\partial k_\theta}{\partial x_i} = -\overline{u_i'\theta}\frac{\partial \bar{T}}{\partial x_i} - \alpha \overline{\frac{\partial \theta}{\partial x_i}\frac{\partial \theta}{\partial x_i}} - \frac{1}{2}\frac{\partial}{\partial x_i}\overline{u_i'\theta^2} + \alpha \frac{\partial^2 k_\theta}{\partial x_j \partial x_j} \tag{1}$$
Identify the different terms in a few words.
4. Simply the previous equation for homogeneous turbulence.
5. The notations $\mathcal{P}_\theta$ and ϵ_θ are introduced to denote the production and the dissipation. To which terms of the equation (1) do they correspond? Provide the units and the sign of these terms.
6. In order to close this equation, the correlation term $-\overline{u_i'\theta}$ must be modeled. Remind the definition of the Prandtl number σ. Following what has been done for Fourier's law and Reynolds' tensor, propose a closure for the term $-\overline{u_i'\theta}$ by introducing the turbulent Prandtl number $\sigma_t = \nu_t/\alpha_t$.
7. What should be the sign of the term $\mathcal{P}_\theta$ in the framework of this model? Comment your answer.

FIGURE 1 – Cumulus clouds : the length scale of the large eddies is about 250 m and the fluctuating velocity is 1 $m.s^{-1}$

Energy dissipated in a cumulus cloud **(4 pts)**

1. Estimate the energy dissipation rate in a cumulus cloud, both per unit mass and for the entire cloud.[2] Compute the total dissipation rate in kilowatts. Also estimate the Kolmogorov scale. Compare with the power received at the surface of the Earth from the Sun.

References

[1] Corrsin, S. and Kistler, A.L., 1954, The free-stream boundaries of turbulent flows, NACA TN-3133, 1-109.

[2] Tennekes, H. & Lumley, J.L., 1972, A first course in turbulence, *The Massachusetts Institute of Technology*, Cambridge.