Turbulence & Instabilités - ECL S7 FLE-a1
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TUTORIAL CLASSES : STUDY OF A FREE SELF-SIMILAR FLOW

Self-similar solution of the mixing layer

The starting point for this exercise is the formal expression of the self-similar solution for the plane mixing layer flow, established in the course slides (you don't have to demonstrate all those results again).

1. Show that for a free plane mixing layer

$$\overline{U}_2 = \Delta U \delta' \left\{ \eta f - \int_0^\eta f \, d\tilde{\eta} \right\}$$

2. From the simplified equation governing the mean flow, show that the self-similarity imposes a linear growth of the mixing layer, denoted $\delta = a(x_1 - x_1^0)$ with a and x_1^0 two constants (x_1^0 represents the virtual spatial origin of the self-similar flow).

3. Show that $S\eta f' = g'$ where S is the spreading rate of the mixing layer

$$S \equiv \frac{u_m}{\Delta U} \delta'$$

4. The Reynolds stress tensor is modeled by introducing a turbulent viscosity ν_t (Boussinesq model), and it will be assumed that $\nu_t = l_m \Delta U$ where $l_m = \alpha \delta(x_1)$ is a mixing length scale. Justify in a few sentences this expression of the turbulent viscosity.

5. Show that the function f must satisfied the following differential equation, $f'' = -S\mathrm{Re}_t \eta f'$, where Re_t denotes a turbulent Reynolds number.

6. By integrating the previous equation, in taking into account the associated boundary conditions, show finally that

$$f(\eta) = \frac{1}{2} \mathrm{erf}\left(\frac{\eta}{\sqrt{2}\sigma} \right) \qquad \text{where} \qquad \mathrm{erf}(\xi) = \frac{2}{\sqrt{\pi}} \int_0^\xi e^{-\zeta^2} d\zeta$$

is the error function.

7. Determine an estimate of the values of S and a the measurements by Champagne *et al.*[2] and by Bell & Mehta[1] (these papers can be downloaded from our website).

To be finish as homework

8. Plot the curve $(\overline{U}_1 - u_m)/\Delta U$ as a function of η and compare to experimental data.

9. Compare also the self-similar solution with the tanh profile used to solve Rayleigh's equation in a previous homework. Include this plot in your copy (homework on Stability).

References

[1] Bell, J.H. & Mehta, J.M., 1990, Development of a two-stream mixing layer from tripped and untripped boundary layers, *AIAA Journal*, **28**(12), 2034-2042.

[2] Champagne, F.H., Pao, Y.H. & Wygnanski, I.J., 1976, On the two-dimensional mixing region *J. Fluid Mech.*, **74**(2), 209-250.

[3] Pope, S.B., 2000, *Turbulent flows*, Cambridge University Press.