Turbulence & Instabilités - ECL S7 FLE-a1
http://acoustique.ec-lyon.fr

Tutorial classes : Vorticity and turbulence

Identification de structures tourbillonnaires

On considère la fimille d'écoulements bidimensionnels définis à partir de la fonction de courant $\psi = (\omega - s)/4x_1^2 + (\omega + s)/4x_2^2$ sur un domaine arbitraire $[-1,1]\times[-1,1]$, et on note $p = p_0$ la pression à l'origine en $x_1 = x_2 = 0$.

1. Rappeler le sens de la fonction de courant, et calculer les composantes du champ de vitesse, de la vorticité, du champ de pression et des tenseurs $\boldsymbol{W}$ et $\boldsymbol{D}$.
2. Pour les trois cas suivants $s = 0$, $\omega = 0$, et $\omega = s$, caractériser et tracer sous Matlab l'écoulement. On calculera en particulier les valeurs propres du tenseur des déformations $\boldsymbol{D}$, et le champ de pression.
3. Discuter ces résultats dans l'optique de pouvoir identifier des structures tourbillonnaires à partir d'images du champ de vitesse. Placer ces écoulements dans un diagramme (D^2, W^2)

Relation of Corrsin & Kistler (1954)

In this exercise, the necessary rotational feature of a turbulent velocity field is emphasized.[1]

1. Remind the definition of an irrotational flow.
2. By considering the following quantity
$$\overline{u_i'\left(\frac{\partial u_i'}{\partial x_j} - \frac{\partial u_j'}{\partial x_i}\right)}$$
demonstrate that
$$\frac{\partial}{\partial x_i}\overline{u_i' u_j'} = \frac{\partial k_t}{\partial x_j}$$
3. Deduce the form of the RANS Equation for the case of a fluctuating irrotational velocity field, and comment carefully your result.
4. Is the Reynolds tensor diagonal for an irrotational flow ?

Oscillating boundary layer

Consider an incompressible boundary layer developing along a flat plate,[2,3] and an imposed freestream (external) velocity given by
$$U_{e1} = U_0\{1 - ax[1 - \cos(2\pi f t)]\}$$
where a is a constant, dimension of inverse length, U_0 is a constant reference velocity and f is the oscillation frequency.

1. Compute the pressure gradient dP_e/dx in the external flow, and its mean value $d\bar{P}_e/dx$ by integrating over one period $T = 1/f$.
2. Compare the expressions obtained for $f = 0$ and for $f \neq 0$. What conclusion can you draw about the separation of the boundary layer ?

References

[1] Corrsin, S. and Kistler, A.L., 1954, The free-stream boundaries of turbulent flows, NACA TN-3133, 1-109.

[2] Fan, S., Lakshminarayana, B. & Barnett, M., 1993, Low-Reynolds-number $k-\epsilon$ model for unsteady turbulent boundary-layer flows, *AIAA Journal*, **31**(10), 1777-1784.

[3] Wilcox, D. C., 1988, Multiscale model for turbulent flows, *AIAA Journal*, **26**(11), 1311-1320.